



MULTIPLE-CHOICE TEST  
in MATHEMATICS

1. (10p) If  $x_1$  and  $x_2$  are the solutions of the equation  $x^2 - (2m - 3)x + m - 1 = 0$ ,  $m \in \mathbf{R}$ , then the value of the expression  $S = x_1 + x_2 - 2x_1x_2$  is:

- a.  $-1$
- b.  $0$
- c.  $m$
- d.  $2m - 3$
- e.  $m - 1$

2. (5p) Let  $f: \mathbf{R} \rightarrow \mathbf{R}$ ,  $f(x) = x^2 + 5x + m + 6$ . The value of  $m \in \mathbf{R}$  such that  $f(x) \geq 0$ ,  $\forall x \in \mathbf{R}$  satisfies:

- a.  $m \in (-1, \frac{1}{8})$
- b.  $m \in [\frac{1}{4}, +\infty)$
- c.  $m < 0$
- d.  $m \in (-1, 0)$
- e.  $m \in \{-1, 0\}$

3. (5p) The sum of the solutions of the equation  $\log_3(x^2 + 3) - \log_3(4x) = 0$  belongs to the interval:

- a.  $(2, 4)$
- b.  $(2, 5)$
- c.  $(4, 6]$
- d.  $[0, 3]$
- e.  $[1, 4)$

4. (10p) If the numbers  $2, a, b$  are in geometric progression and the numbers  $2, 9, a$  are in arithmetic progression, then  $\sqrt{a + b}$  is:

- a.  $12$
- b.  $12\sqrt{2}$
- c.  $14$
- d.  $11\sqrt{2}$
- e.  $13$

5. (10p) If  $A = \begin{pmatrix} m & m+1 & m+2 \\ n & n+1 & n+2 \\ 1 & 1 & m \end{pmatrix} \in M_3(\mathbf{R})$  and  ${}^tA$  is the transpose of  $A$ , then:

- a.  $\text{rank } A < 2$
- b.  $\det(A - {}^tA) = -1$
- c.  $\det(A - {}^tA) = 0$
- d.  $\det(A - {}^tA) = 1$
- e.  $\text{rank } A = 1$

6. (10p) Consider the system



$$\begin{cases} (a-2)x - 2y + z = 1 \\ x + y + z = a - 2 \\ (a-1)x - 2y + 2z = 1 \end{cases}, \quad \text{where } a \in \mathbb{R}.$$

The system is compatible determined (that is, it has a unique solution) for:

- a.  $a \in \mathbb{R} \setminus \{3\}$
- b.  $a = 3$
- c.  $a \in \{-3, 3\}$
- d.  $a \in (-\infty, 0)$
- e.  $a \in [-1, 1]$

7. (10p) If  $L = \lim_{n \rightarrow +\infty} \frac{4^{-3n} - 4^{9n}}{4^{-n} - 4^{3n}}$  then:

- a.  $L = \ln 4$
- b.  $L = e$
- c.  $L = 0$
- d.  $L = +\infty$
- e.  $L = 1$

8. (10p) Let  $f: (0, +\infty) \rightarrow \mathbb{R}$ , defined by

$$f(x) = \frac{\ln x}{x}.$$

Then its derivative  $f'(x)$  is equal to:

- a.  $\frac{\ln x}{x^2}$
- b.  $\frac{1 + \ln x}{x^2}$
- c.  $\frac{1 + x \ln x}{x^2}$
- d.  $\frac{1 - \ln x}{x^2}$
- e.  $\frac{1 - x \ln x}{x^2}$

9. (10p) Let  $a, b, c \in \mathbb{R}$ . If  $F: \mathbb{R} \rightarrow \mathbb{R}, F(x) = 2ae^{bx^2+b} + 3cx^3$  is a primitive of the function  $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2xe^{x^2+1} + 3x^2$  then the value of the expression  $S = e^{b-1} - \log_6(ac)$  is:

- a. 2
- b. 0
- c. -1
- d.  $e + 1$
- e.  $1 - e$

10. (10p) Let  $I_n = \int_1^2 x^{n-\frac{1}{2}} e^x dx, n \in \mathbb{N} \setminus \{0\}$ . Then, for all  $n \in \mathbb{N} \setminus \{0, 1\}$  the expression  $S = I_n + \frac{2n-1}{2} I_{n-1}$  is equal to:

- a.  $e - 2^{n-\frac{1}{2}} e^2$
- b.  $e \left( 2^{n+\frac{1}{2}} - e \right)$
- c.  $e \left( 2^{n+\frac{1}{2}} e - 1 \right)$
- d.  $e 2^n + e^2$
- e.  $2^{n-\frac{1}{2}} e^2 - e$



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2. (5p) The sum of the solutions of the equation  $\log_3(x^2 + 3) - \log_3(4x) = 0$  belongs to the interval:

- a.  $(4, 6]$
- b.  $[0, 3]$
- c.  $(2, 5)$
- d.  $[1, 4)$
- e.  $(2, 4)$

3. (10p) Let  $I_n = \int_1^2 x^{n-\frac{1}{2}} e^x dx$ ,  $n \in \mathbf{N} \setminus \{0\}$ . Then, for all  $n \in \mathbf{N} \setminus \{0, 1\}$  the expression  $S = I_n + \frac{2n-1}{2} I_{n-1}$  is equal to:

- a.  $e \left( 2^{n+\frac{1}{2}} e - 1 \right)$
- b.  $e - 2^{n-\frac{1}{2}} e^2$
- c.  $2^{n-\frac{1}{2}} e^2 - e$
- d.  $e \left( 2^{n+\frac{1}{2}} - e \right)$
- e.  $e 2^n + e^2$

4. (10p) If  $x_1$  and  $x_2$  are the solutions of the equation  $x^2 - (2m - 3)x + m - 1 = 0$ ,  $m \in \mathbf{R}$ , then the value of the expression  $S = x_1 + x_2 - 2x_1x_2$  is:

- a.  $2m - 3$
- b. 0
- c.  $m - 1$
- d.  $m$
- e. -1

5. (10p) If the numbers 2,  $a$ ,  $b$  are in geometric progression and the numbers 2, 9,  $a$  are in arithmetic progression, then  $\sqrt{a+b}$  is:

- a.  $12\sqrt{2}$
- b. 14
- c. 13
- d. 12



e.  $11\sqrt{2}$

6. (10p) Consider the system

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- c.  $e + 1$
- d.  $1 - e$
- e.  $2$

8. (10p) If  $A = \begin{pmatrix} m & m+1 & m+2 \\ n & n+1 & n+2 \\ 1 & 1 & m \end{pmatrix} \in M_3(\mathbf{R})$  and  ${}^tA$  is the transpose of  $A$ , then:

- a.  $\text{rank } A = 1$
- b.  $\det(A - {}^tA) = -1$
- c.  $\det(A - {}^tA) = 0$
- d.  $\text{rank } A < 2$
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9. (10p) If  $L = \lim_{n \rightarrow +\infty} \frac{4^{-3n} - 4^{9n}}{4^{-n} - 4^{3n}}$  then:

- a.  $L = 0$
- b.  $L = +\infty$
- c.  $L = 1$
- d.  $L = e$
- e.  $L = \ln 4$

10. (5p) Let  $f: \mathbf{R} \rightarrow \mathbf{R}, f(x) = x^2 + 5x + m + 6$ . The value of  $m \in \mathbf{R}$  such that  $f(x) \geq 0, \forall x \in \mathbf{R}$  satisfies:

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- a. 0
- b. -1
- c. 2
- d.  $1 - e$
- e.  $e + 1$

5. (10p) Let  $I_n = \int_1^2 x^{n-\frac{1}{2}} e^x dx, n \in \mathbf{N} \setminus \{0\}$ . Then, for all  $n \in \mathbf{N} \setminus \{0, 1\}$  the expression  $S = I_n + \frac{2n-1}{2} I_{n-1}$  is equal to:

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d.  $e\left(2^{n+\frac{1}{2}}e - 1\right)$

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6. (10p) If  $L = \lim_{n \rightarrow +\infty} \frac{4^{-3n} - 4^{9n}}{4^{-n} - 4^{3n}}$  then:

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- d.  $e 2^n + e^2$



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- c.  $a \in \{-3, 3\}$
- d.  $a \in \mathbf{R} \setminus \{3\}$
- e.  $a = 3$

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- b.  $12\sqrt{2}$
- c. 13
- d.  $11\sqrt{2}$
- e. 12

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- d.  $\frac{\ln x}{x^2}$
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a.  $\text{rank } A = 1$   
b.  $\det(A - {}^t A) = 0$   
c.  $\text{rank } A < 2$   
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- d. 13
- e. 14

5. (10p) If  $x_1$  and  $x_2$  are the solutions of the equation  $x^2 - (2m - 3)x + m - 1 = 0, m \in \mathbb{R}$ , then the value of the expression  $S = x_1 + x_2 - 2x_1x_2$  is:

- a.  $2m - 3$
- b.  $m$
- c. 0
- d. -1
- e.  $m - 1$

6. (10p) Let  $f: (0, +\infty) \rightarrow \mathbb{R}$ , defined by

$$f(x) = \frac{\ln x}{x}.$$



Then its derivative  $f'(x)$  is equal to:

- a.  $\frac{1+x\ln x}{x^2}$
- b.  $\frac{1-\ln x}{x^2}$
- c.  $\frac{1+\ln x}{x^2}$
- d.  $\frac{1-x\ln x}{x^2}$
- e.  $\frac{\ln x}{x^2}$

7. (10p) Consider the system

$$\begin{cases} (a-2)x - 2y + z = 1 \\ x + y + z = a - 2 \\ (a-1)x - 2y + 2z = 1 \end{cases}, \quad \text{where } a \in \mathbb{R}.$$

The system is compatible determined (that is, it has a unique solution) for:

- a.  $a \in (-\infty, 0)$
- b.  $a = 3$
- c.  $a \in \{-3, 3\}$
- d.  $a \in [-1, 1]$
- e.  $a \in \mathbb{R} \setminus \{3\}$

8. (10p) Let  $I_n = \int_1^2 x^{n-\frac{1}{2}} e^x dx$ ,  $n \in \mathbb{N} \setminus \{0\}$ . Then, for all  $n \in \mathbb{N} \setminus \{0, 1\}$  the expression  $S = I_n + \frac{2n-1}{2} I_{n-1}$  is equal to:

- a.  $e 2^n + e^2$
- b.  $e - 2^{n-\frac{1}{2}} e^2$
- c.  $2^{n-\frac{1}{2}} e^2 - e$
- d.  $e \left( 2^{n+\frac{1}{2}} e - 1 \right)$
- e.  $e \left( 2^{n+\frac{1}{2}} - e \right)$

9. (5p) The sum of the solutions of the equation  $\log_3(x^2 + 3) - \log_3(4x) = 0$  belongs to the interval:

- a.  $(4, 6]$
- b.  $[0, 3]$
- c.  $(2, 4)$
- d.  $[1, 4)$
- e.  $(2, 5)$

10. (10p) Let  $a, b, c \in \mathbb{R}$ . If  $F: \mathbb{R} \rightarrow \mathbb{R}, F(x) = 2ae^{bx^2+b} + 3cx^3$  is a primitive of the function  $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2xe^{x^2+1} + 3x^2$  then the value of the expression  $S = e^{b-1} - \log_6(ac)$  is:

- a. 0
- b.  $e + 1$
- c. -1
- d. 2
- e.  $1 - e$